Problem 2 – Hands of a clock

At mid-day the hour hand and minute hand of a clock are exactly one above the other.

At what time are they next together?

Minute hand moves at 6°/min
Hour hand moves at $\frac{1}{2}^\circ$/min.

Minute hand closes the gap between itself and hour hand at a rate of $5\frac{1}{2}^\circ$/min.

The gap between them at 12 noon is 360°.

The time taken to close the gap is:

$$\frac{360^\circ}{5\frac{1}{2}} = 65\frac{5}{11} \text{ mins.}$$

Minute hand will have caught the hour hand at $5\frac{5}{11}$ mins past 1.

hour hand: speed = $\frac{1}{2}^\circ$ / min = $\omega = \theta/t$

$\Rightarrow \frac{1}{2} = \theta/t$

$\Rightarrow \frac{t}{2} = \theta$. — (1)

Minute hand: speed = 6°/min = $\omega = \theta/t$

$\Rightarrow 6 = \frac{\theta + 360}{t}$

$\Rightarrow 6t = \theta + 360$. — (2)

Solving (1) + (2)

$6t = \frac{t}{2} + 360$

$5\frac{1}{2}t = 360$

$t = \frac{360}{5\frac{1}{2}} = \frac{720}{11}$

$t = 65\frac{5}{11}$ mins.

hands meet at $5\frac{5}{11}$ mins part 1.

$$_{M}\omega = 360^\circ/\text{hr} = 6^\circ/\text{Min}.$$

$$_{H}\omega = 30^\circ/\text{hr} = \tfrac{1}{2}^\circ/\text{Min}.$$

$$_{M}\omega_{H} = (6 - \tfrac{1}{2})^\circ/\text{Min} = 5\tfrac{1}{2}^\circ/\text{Min}.$$

Time taken to be together again $= \dfrac{360}{5\frac{1}{2}}$ mins

$= 65\ 5/11$ mins.

Hands will be together again at $5\ 5/11$ mins past 1.